Exam 2B Solutions

Curve

If x is your raw grade and y is the curved grade, then

$$y = \begin{cases} 0.8x + 36 & \text{if } x \geq 70 \\ 0.6x + 50 & \text{if } 35 < x < 70 \\ 2x + 1 & \text{if } x \leq 35 \end{cases}$$

Problem 1: Card Probabilities

Part 1A: What is the probability of getting exactly 2 black cards in a 5-card hand from a standard deck of 52 cards?

- **Step 1:** Calculate the number of ways to choose exactly 2 black cards out of the 26 available black cards: $C(26,2)$.
- **Step 2:** Calculate the number of ways to choose the remaining 3 cards from the 26 red cards: $C(26,3)$.
- **Step 3:** Divide the product by the total number of possible 5-card hands: $C(52,5)$.
- **Answer:** $\boxed{\frac{C(26,2) \times C(26,3)}{C(52,5)}}$

Part 1B: What is the probability of getting at least 2 black cards in a 5-card hand from a standard deck of 52 cards?

- **Step 1:** "At least 2" means the hand can have exactly 2, 3, 4, or 5 black cards.
- **Step 2:** Sum the combinations for each of these mutually exclusive scenarios and divide by the total combinations.
- **Answer:** $\boxed{\frac{C(26,2)C(26,3) + C(26,3)C(26,2) + C(26,4)C(26,1) + C(26,5)C(26,0)}{C(52,5)}}$
- Alternatively, we can calculate the complement probability instead:
- **Alternate Answer:** $\boxed{1 - \frac{C(26,0)C(26,5)}{C(52,5)} - \frac{C(26,1)C(26,4)}{C(52,5)}}$

Part 1C: What is the probability of getting exactly 2 black cards and 2 Ace cards in a 5-card hand from a standard deck of 52 cards?

Note: Generous partial credit is given since this problem is too difficult (involves too many cases).

- **Step 1:** Partition the deck into four sets: Black Aces (2), Red Aces (2), Black non-Aces (24), and Red non-Aces (24).
- **Step 2:** Account for the three ways this hand can be built:
    1. *2 Black Aces:* Choose 2 Black Aces, 0 Red Aces, 0 Black non-Aces, 3 Red non-Aces $\rightarrow C(2,2)C(2,0)C(24,0)C(24,3)$
    2. *1 Black Ace & 1 Red Ace:* Choose 1 Black Ace, 1 Red Ace, 1 Black non-Ace, 2 Red non-Aces $\rightarrow C(2,1)C(2,1)C(24,1)C(24,2)$
    3. *2 Red Aces:* Choose 0 Black Aces, 2 Red Aces, 2 Black non-Aces, 1 Red non-Ace $\rightarrow C(2,0)C(2,2)C(24,2)C(24,1)$
- **Answer:** $\boxed{\frac{C(2,2)C(24,3) + C(2,1)C(2,1)C(24,1)C(24,2) + C(2,2)C(24,2)C(24,1)}{C(52,5)}}$

Part 1D: What is the probability of getting exactly 2 black cards or 2 Ace cards in a 5-card hand from a standard deck of 52 cards?

- **Step 1:** Use the Principle of Inclusion-Exclusion: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- **Step 2:** Subtract the intersection (Problem 1C) from the sum of the individual probabilities (Problem 1A and exactly 2 Aces).
- **Answer:** $\boxed{\frac{C(26,2)C(26,3) + C(4,2)C(48,3)}{C(52,5)} - \text{(Answer to Part 1C)}}$

Problem 2: Dice

Part 2A: A fair dice is rolled six times. Find the probability that the first five rolls have no 2's, and the sixth roll is a 2.

- **Step 1:** The probability of not rolling a 2 on a fair die is $5/6$. The probability of rolling a 2 is $1/6$.
- **Step 2:** Multiply the independent probabilities.
- **Answer:** $\boxed{(5/6)^5 \times (1/6)}$

Part 2B: A fair dice is rolled six times. Find the probability that exactly three of the six rolls come up as either 1's or 2's.

- **Step 1:** The probability of success (rolling a 1 or 2) is $2/6 = 1/3$.
- **Step 2:** Use the binomial probability formula for exactly 3 successes in 6 trials.
- **Answer:** $\boxed{C(6,3) \times (1/3)^3 \times (2/3)^3}$

Problem 3: Router Connections

To connect to Google, your ISP uses 2 available routers with success probabilities $p_1 = 0.8$ and $p_2 = 0.9$.

Part 3A: Find the probability that no router successfully connects you to Google.

- **Step 1:** Calculate the failure probabilities for each router: $1 - 0.8 = 0.2$ and $1 - 0.9 = 0.1$.
- **Step 2:** Multiply the independent failure probabilities.
- **Answer:** $\boxed{0.2 \times 0.1}$ (or 0.02)

Part 3B: Find the probability that at least one router successfully connects you to Google.

- **Step 1:** Use the complement rule: subtract the probability of "no success" from 1.
- **Answer:** $\boxed{1 - (0.2 \times 0.1)}$ (or 0.98)

Problem 4: Multiple-Choice Test

A test has 5 multiple-choice questions with 4 options each. A student randomly selects an option with equal probability. Find the probability that the student gets at least 2 questions correct.

- **Step 1:** The probability of guessing correctly is $1/4$.
- **Step 2:** "At least 2" is the complement of getting exactly 0 or exactly 1 correct.

- **Answer:** $\boxed{1 - [C(5,0)(1/4)^0(3/4)^5 + C(5,1)(1/4)^1(3/4)^4]}$

Problem 5: Conditional Probabilities

Problem 5A: A fair dice is rolled twice. What is the probability that the sum of the two rolls is ≤ 4, given that the first roll is a 1?

- **Step 1:** If the first roll is a 1, the second roll must be 1, 2, or 3 to keep the sum ≤ 4.
- **Step 2:** There are 3 successful outcomes out of 6 possible outcomes for the second die.
- **Answer:** $\boxed{3/6}$

Problem 5B: What is the probability that the first roll is a 1, given that the sum of the two rolls is ≤ 4?

- **Step 1:** List all sums ≤ 4: (1,1), (1,2), (1,3), (2,1), (2,2), (3,1). Total = 6 outcomes.
- **Step 2:** Identify outcomes where the first roll is 1: (1,1), (1,2), (1,3). Total = 3 outcomes.
- **Answer:** $\boxed{3/6}$

Problem 5C: A bag has one fair coin and one unfair coin (both heads). One is randomly grabbed and flipped three times. Given three heads, what is the probability it was the unfair coin?

- **Step 1:** Use Bayes' Theorem. $P(U) = 1/2$, $P(F) = 1/2$.
- **Step 2:** $P(3H \mid U) = 1$. $P(3H \mid F) = (1/2)^3 = 1/8$.
- **Step 3:** $P(U \mid 3H) = \dfrac{P(3H \mid U)P(U)}{P(3H \mid U)P(U) + P(3H \mid F)P(F)}$.
- **Answer:** $\boxed{\dfrac{1 \times 0.5}{1 \times 0.5 + 0.125 \times 0.5}}$ (or $8/9$)

Problem 6 (~Lesson 8, Problem 5)

Typo: In the problem statement, "math or history" should be changed to "math or CS".

Part 6A: The student studies math and CS.

- **Step 1:** Use the union formula: $|M \cap CS| = |M| + |CS| - |M \cup CS| = 40 + 60 - 80 = 20$.
- **Answer:** $\boxed{20/100}$

Part 6B: The student studies neither subject.

- **Step 1:** Subtract the union from the total pool of 100 students.
- **Answer:** $\boxed{(100 - 80)/100}$

Part 6C: The student studies CS but not math.

- **Step 1:** Subtract the intersection from the total number of CS students: $60 - 20 = 40$.
- **Answer:** $\boxed{40/100}$

Part 6D: The student studies math but not CS.

- **Step 1:** Subtract the intersection from the total number of math students: $40 - 20 = 20$.
- **Answer:** $\boxed{20/100}$

Problem 7: Netflix User Segmentation (~Lesson 9 Exercise 1)

Using the provided table data: Total Male = 60, Total Female = 70.

Part 7A: Find the probability that a person is male, given that the person is a 0-19.

- Total 0-19 users = $10 \text{ (male)} + 20 \text{ (female)} = 30$.
- **Answer:** $\boxed{10/30}$

Part 7B: Find the probability that a person is a 0-19, given that the person is male.

- There are 60 males total, of which 10 are in the 0-19 group.
- **Answer:** $\boxed{10/60}$

Part 7C: Find the probability that a person is not 30+, given that the person is female.

- "Not 30+" females are the 0-19 and 20-29 groups. $20 + 30 = 50$.
- **Answer:** $\boxed{50/70}$

Problem 8: Operator Training (~Lesson 12 Problem 1)

Operators who attend training meet quotas 80% of the time, and those who don't meet it 60% of the time. Fifty percent of new operators attend training.

- **Step 1:** Given a new operator meets their quota (Q), find the probability they attended training (T).
- **Step 2:** Use Bayes' Theorem: $P(T \mid Q) = \dfrac{P(Q \mid T)P(T)}{P(Q \mid T)P(T) + P(Q \mid T')P(T')}$.
- **Answer:** $\boxed{\dfrac{0.8 \times 0.5}{0.8 \times 0.5 + 0.6 \times 0.5}}$

Problem 9: CPU Testing(~Lesson 11 Problem 2)

Every 3rd CPU is tested (so the probability of being tested is $1/3$). Tests run three independent tasks with failure rates of 0.01, 0.02, and 0.03.

Part 9A: Find the probability that a CPU was tested AND failed some test.

The probability of failing *at least one* test is the complement of passing all three: $1 - (0.99 \times 0.98 \times 0.97)$.

Answer: $\boxed{(1/3) \times (1 - (0.99 \times 0.98 \times 0.97))}$

Part 9C: Given that a CPU was tested, what is the probability that it failed tasks 1 or 2?

Since the CPU is already being tested, we calculate $P(F_1 \cup F_2) = P(F_1) + P(F_2) - P(F_1 \cap F_2)$, giving:

$$\boxed{0.01 + 0.02 - (0.01 \times 0.02)}$$

Problem 10: Max Roll

A fair dice is rolled five times. What is the probability that the largest number rolled in those five rolls is a 3?

- **Step 1:** This implies all 5 rolls are ≤ 3, but *not* all 5 rolls are ≤ 2.
- **Step 2:** Subtract the probability of rolling all 1s and 2s from the probability of rolling only 1s, 2s, and 3s.
- **Answer:** $\boxed{(3/6)^5 - (2/6)^5}$

Problem 11

A family has two children. One is a boy born on a weekend. What is the probability that both children are boys?

- **Step 1:** The reduced sample space is a union of event A where Boy 1 is born on a weekend and event B where Boy 2 is born on a weekend. Here A has 2 outcomes for Boy 1 (Saturday or Sunday) times any possibility for Child 2 (B/G times day of week = $2 \times 7 = 14$). Overall $|A| = 2 \times 14 = 28$. Likewise $|B| = 28$. But $A \cap B|$ consist of outcomes where both children are boys born on weekends, which allows 4 possibilities. So $|A \cup B| = |A| + |B| - |A \cap B| = 28 + 28 - 4 = 54$.
- **Step 2:** The desired outcomes is where both children are boys. Since we are working in the reduced sample space $A \cup B$, the set of desired outcomes is the union of event A' where first child is a boy born on a weekend and second child is a boy born on any day of the week and event B' where second child is a boy born on a weekend and first child is a boy born on any day of the week. Observe that $|A'| = |B'| = 14$ and $|A' \cap B'| = 4$ so $|A' \cup B'| = 14 + 14 - 4 = 24$.
- **Answer:** The resulting conditional probability is the answer for Step 2 divided by the answer for Step 1, or $24/54$.